11 Introduction to Three Dimensional Geometry

If you're a tech enthusiast, you've probably heard of the term 'Virtual Reality' (VR). As the name suggests, it creates non-physical materiality that is very similar to reality. 3D VR is created or produced digitally. 3D environments are created by using computer software and artificial intelligence with the help of three dimensional geometry to replicate the real world or to create entirely new environments. These environments, like a game, can be thoroughly explored by the user. Users have the ability to interact with the environment, avatars, and items.

Topic Notes

- *Coordinate Planes in 3-D Space*

COORDINATE PLANES IN 3-D SPACE 1

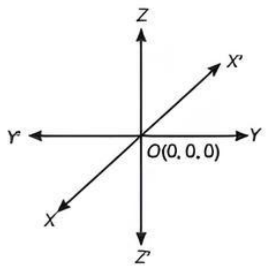
TOPIC 1

COORDINATE AXES

In the previous chapter, we have discussed that in order to locate the position of a point in a plane, we need to fix mutually perpendicular lines in the plane and two real numbers, which are the distances of the point from these lines.

These lines are called coordinate axes and the two numbers are called coordinates of the point with respect to the axes. Similarly for 3 axes, we require three real numbers which are the distances of the points from the two adjacent walls. These three numbers are called the coordinates of the point. So, a point in space has three coordinates.

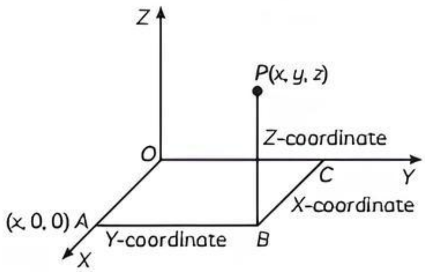
Let XOX', YOY' and ZOZ' be the three mutually perpendicular lines, intersecting at O. The point $O(0, 0, 0)$ is called the origin and the lines (XOX', YOY' and ZOZ') are called rectangular coordinate axes say X, Y and Z, respectively. Thus in the given figure, $X'OX$ is called the X-axis, $Y'OY$ is called the Y-axis, and $Z'OZ$ is called the Z-axis.



3-dimensional geometry involves the mathematics of shapes in 3D space and involves 3 coordinates in the XYZ plane which are x-coordinate, y-coordinate, and z-coordinate. The shapes that occupy space are called 3D shapes. 3D shapes can also be defined as solid shapes having three dimensions length, width, and height.

Coordinate of a Point in Space

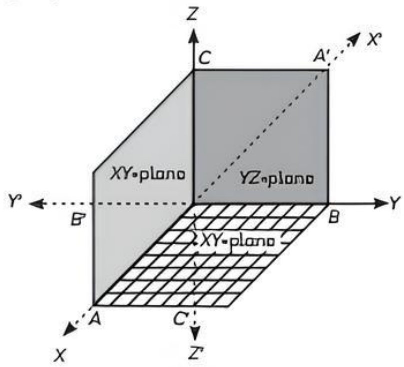
Thus, for each point in space there exists an ordered 3-tuple of real numbers for its representation.



In the figure the co-ordinates of P are given by (x, y, z). The coordinates of the origin O is $(0, 0, 0)$. Also the coordinates of the point A is given by $(x, 0, 0)$ as A lies completely on the x-axis.

Coordinate Axes and Coordinate Planes in Space

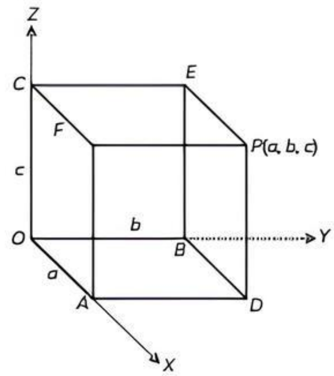
The three coordinate axes determine the three coordinate planes illustrated in figure. The xy-plane is the plane that contains the x and y-axes; the yz-plane contains the y and z-axes; the xz-plane contains the x and z-axes. These three coordinate planes divide space into eight parts, called octants.



The following table shows the signs of coordinates of points in various octants:

Octants / Coordinates	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Example 1.1: In the given figure, if the coordinates of point P are (a, b, c), then write the coordinate of A, D, B, C and E.



Ans. Given, the coordinates of point P are (a, b, c).

Which shows that, $OA = a$, $OB = b$ and $OC = c$.

Now, point A lies on X-axis, so its coordinates are $(a, 0, 0)$.

Point D lies in XY-plane, so its coordinates are $(a, b, 0)$.

Point B lies on Y-axis, so its coordinates are $(0, b, 0)$.

Point C lies on Z-axis, so its coordinates are $(0, 0, c)$ and point E lies in YZ-plane, so its coordinates are $(0, b, c)$.

Hence, the coordinates of required points are

$A(a, 0, 0)$, $D(a, b, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $E(0, b, c)$.

Example 1.2: What are the coordinates of the vertices of a cube whose edge is 5 units, one of whose vertices coincides with the origin and three edges passing through the origin coincides with the positive direction of the axes through the origin?

Ans. Given, edge of a cube is 5 unit. It is clear that

Coordinate of $O = (0, 0, 0)$

Coordinate of $G = (0, 5, 0)$

Coordinate of $B = (5, 5, 0)$

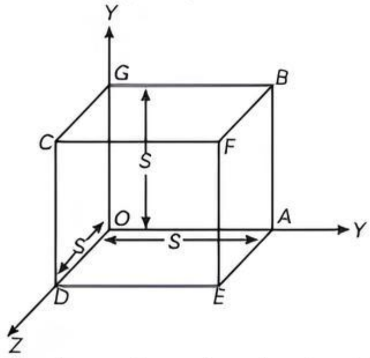
Coordinate of $E = (5, 0, 5)$

Coordinate of $A = (5, 0, 0)$

Coordinate of $D = (0, 0, 5)$

Coordinate of $F = (5, 5, 5)$

Coordinate of $C = (0, 5, 5)$



If a point P lies in xy-plane, then by the definition of coordinates of a point, z-coordinate of P is zero. Therefore, the coordinates of a point on xy-plane are of the form $(x, y, 0)$ and we may take the equation of xy-plane as $z = 0$. Similarly, the coordinates of any point in yz and zx-planes are of the forms $(0, y, z)$ and $(x, 0, z)$ respectively and their equations may be taken as $x = 0$ and $y = 0$ respectively.

If a point lies on the x-axis, then its y and z-coordinates are both zero. Therefore, the coordinates of a point on x-axis are of the form $(x, 0, 0)$ and we may take the equation of x-axis as $y = 0$, $z = 0$. Similarly, the coordinates of a point on y and z-axes are of the form $(0, y, 0)$ and $(0, 0, z)$ respectively and their equations may be taken as $x = 0$, $z = 0$ and $x = 0$, $y = 0$ respectively.

We have the following working rule:

Step I: Using first two coordinates of the point, find quadrant number.

Step II: If third coordinates is positive, then octant number = quadrant number.

Step III: If third coordinate is negative, then octant number = quadrant number + 4.

Remarks:

(1) The coordinates of a point lying on x-axis are of the form $(x, 0, 0)$.

(2) The coordinates of a point lying on y-axis are of the form $(0, y, 0)$.

(3) The coordinates of a point lying on z-axis are of the form $(0, 0, z)$

(4) The coordinates of a point lying on yz-plane are of the form $(0, y, z)$.

(5) The coordinates of a point lying on xz-plane are of the form $(x, 0, z)$.

(6) The coordinates of a point lying on xy-plane are of the form $(x, y, 0)$.

Example 1.3: Name the octants in which the following points lie:

(A) (1, 2, 3) **(B) (–4, 2, 5)**

(C) (–3, –1, 6) **(D) (4, –2, 5)**

(E) (4, 2, –5) **(F) (–4, 2, –5)**

(G) (–2, –4, –7) **(H) (4, –2, –5)**

Ans. **(A)** I octant

(B) II octant

(C) III octant

(D) IV octant

(E) V octant

(F) VI octant

(G) VII octant

(H) VIII octant

TOPIC 2

DISTANCE BETWEEN TWO POINTS

The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

If O is the origin and $P(x, y, z)$ is a point in space, then

$$OP = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

Example 1.4. Find the coordinates of a point on *Y*-axis which is at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$. **[NCERT]**

Ans. Let the point be $A(0, y, 0)$.

Given point is $P(3, -2, 5)$

$$AP = 5\sqrt{2}$$

The distance between the points *A* and *P*

$$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$\Rightarrow \quad \sqrt{(0-3)^2+(y+2)^2+(0-5)^2} = 5\sqrt{2}$$

$$\Rightarrow \quad 9 + (y+2)^2 + 25 = 50$$

$$\Rightarrow \quad (y+2)^2 = 16$$

$$\Rightarrow \quad y + 2 = \pm 4$$

$$\Rightarrow \quad y = 2, -6$$

Hence, the coordinates of the required point are $(0, 2, 0)$ and $(0, -6, 0)$.

Example 1.5. Find the distance between the points $P(2, 3, 5)$ and $Q(4, 3, 1)$. **[NCERT]**

Ans. Given points are $P(2, 3, 5)$ and $Q(4, 3, 1)$.

Then, the distance between *P* and *Q* is given by,

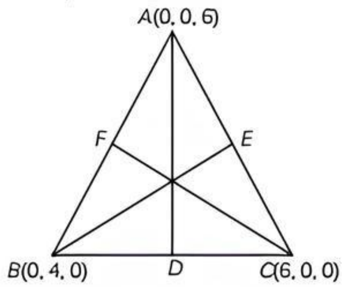
$$PQ = \sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$

$$= \sqrt{4+0+16}$$

$$= \sqrt{20} \text{ units}$$

Example 1.6. Find the lengths of the medians of the triangle with vertices $A(0, 0, 6)$, $B(0, 4, 0)$ and $C(6, 0, 0)$. **[NCERT]**

Ans. Let *D*, *E* and *F* be the mid-points of sides, *BC*, *CA* and *AB* respectively.



The coordinates of *D*, *E* and *F* are

$$D\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$$

$$E\left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = (3, 0, 3)$$

and $$F\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) = (0, 2, 3)$$

$$\therefore \quad AD = \sqrt{(0-3)^2+(0-2)^2+(6-0)^2}$$

$$= \sqrt{9+4+36} = \sqrt{49} = 7$$

$$BE = \sqrt{(0-3)^2+(4-0)^2+(0-3)^2}$$

$$= \sqrt{9+16+9} = \sqrt{34}$$

$$CF = \sqrt{(6-0)^2+(0-2)^2+(0-3)^2}$$

$$CF = \sqrt{36+4+9} = \sqrt{49} = 7$$

Example 1.7: If *A* and *B* be the point (3, 4, 5) and (–1, 3, –7), respectively, find the equation of the set of points *P* such that $PA^2 + PB^2 = k^2$, where *k* is constant. **[NCERT]**

Ans. The coordinates of point *A* and *B* are given as (3, 4, 5) and (–1, 3, –7) respectively. Let the coordinates of point *P* be (x, y, z).

On using distance formula, we get

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$

$$= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 5^2 - 10z$$

$$= x^2 + y^2 + z^2 - 6x - 8y - 10z + 50$$

$$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$$

$$= x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z$$

$$= x^2 + y^2 + z^2 + 2x - 6y + 14z + 59$$

Now, if $PA^2 + PB^2 = k^2$, then

$$(x^2 + y^2 + z^2 - 6x - 8y - 10z + 50) + (x^2 + y^2 + z^2 + 2x - 6y + 14z + 59) = k^2$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$

$$\Rightarrow \quad 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) + 109 = k^2$$

$$\Rightarrow \quad 2(x^2 + y^2 + z^2 - 2x - 7y + 2z) = k^2 - 109$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$$

Thus, the required equation is

$$x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2-109}{2}$$

Example 1.8: Find the equation of the set of points *P*, the sum of whose distances from $A(4, 0, 0)$ and $B(-4, 0, 0)$ is equal to 10. **[NCERT]**

Ans. Let $P(x, y, z)$ be the point

$$\therefore \quad PA + PB = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2+(y-0)^2+(z-0)^2} + \sqrt{(x+4)^2+(y-0)^2+(z-0)^2} = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2+y^2+z^2} + \sqrt{(x+4)^2+y^2+z^2} = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2+y^2+z^2} = 10 - \sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad (x-4)^2+y^2+z^2 = 100 - 20\sqrt{(x+4)^2+y^2+z^2} + (x+4)^2 + y^2 + z^2$$

$$\Rightarrow \quad (x-4)^2 - (x+4)^2 - 100 = -20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad x^2 - 8x + 16 - x^2 - 8x - 16 - 100$$

$$= -20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad (4x+25) = 5\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow \quad (4x+25)^2 = 25(x+4)^2 + 25y^2 + 25z^2$$

$$\Rightarrow \quad 16x^2 + 200x + 625 = 25x^2 + 200x + 400 + 25y^2 + 25z^2$$

$$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 = 225$$

Example 1.9: Verify the following

(A) (0, 7, –10), (1, 6, –6) and (4, 9, –6) are the vertices of an isosceles triangle.

(B) (0, 7, 10), (–1, 6, 6) and (–4, 9, 6) are the vertices of a right-angled triangle. [NCERT]

Ans. (A) Let the given points be $A(0, 7, -10)$, $B(1, 6, -6)$ and $C(4, 9, -6)$.

$$= \sqrt{(0-1)^2+(7-6)^2+(-10+6)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

$$= \sqrt{(1-4)^2+(6-9)^2+(-6+6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$\therefore AB = BC$, so ΔABC is isosceles triangle.

(B) Let the given points be $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$.

$$AB = \sqrt{(0+1)^2+(7-6)^2+(10-6)^2}$$

$$= \sqrt{1+1+16} = \sqrt{18}$$

$$BC = \sqrt{(-1+4)^2+(6-9)^2+(6-6)^2}$$

$$= \sqrt{9+9+0} = \sqrt{18}$$

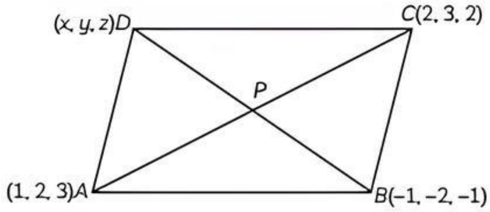
$$CA = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36}$$

Since, $AB^2 + BC^2 = CA^2$ so, ΔABC is right-angled triangle.

Example 1.10: If the vertices of a parallelogram *ABCD* are $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$, then find the fourth vertex *D*. [NCERT]

Ans. Let the fourth vertex of the parallelogram *ABCD* is $D(x, y, z)$. Then, the mid-point of *AC* is *P*.



$$P\left(\frac{1+2}{2}, \frac{2+3}{2}, \frac{3+2}{2}\right) \text{ i.e., } P\left(\frac{3}{2}, \frac{5}{2}, \frac{5}{2}\right)$$

Mid-point of *BD* is also *P* (as diagonal bisect each other)

$$\frac{3}{2} = \frac{-1+x}{2}$$

$$\Rightarrow \quad x = 4$$

$$\frac{5}{2} = \frac{-2+y}{2}$$

$$\Rightarrow \quad y = 7$$

$$\frac{5}{2} = \frac{-1+z}{2}$$

$$\Rightarrow \quad z = 6$$

$\therefore$ The coordinates of fourth vertex is (4, 7, 6).

Example 1.11: Case Based:

Four students in traditional dresses represent four states of India, standing at the points represented by O(0, 0, 0), A(a, 0, 0), B (0, b, 0) C(0, 0, 0). If a girl representing BHARATMATA be placed in such a way that she is equidistant from the four students, then answer the following questions which are based on above it.

Based on the above information, answer the following questions:

(A) Find the *x*-coordinate of girl representing BHARATMATA.

(B) Find the *y*-coordinate of girl representing BHARATMATA.

(C) *z*-coordinate of girl representing BHARATMATA is:

(a) b (b) c

(c) $\frac{c}{2}$ (d) $2c$

(D) Assertion (A): The concept used for finding the coordinates of point is distance formula.

Reason (R): The octants in which (2, 3, 4) lies is I octant.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

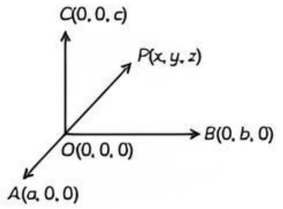
(d) (A) is false but (R) is true.

(E) Which of the following is coordinates of origin point?

(a) (0, 0, 0) (b) (0, *b*, 0)

(c) (*a*, 0, 0) (d) (0, 0, *c*)

Ans. (A) Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.



Then, $PA = PB = PC = OP$

Now, $OP = PA$

$\Rightarrow \quad OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = (x - a)^2 + (y - 0)^2 + (z - 0)^2$

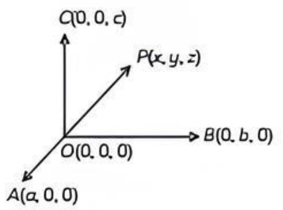
$[\because \text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 + z_2)^2}]$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + a^2 - 2ax + y^2 + z^2$

$\Rightarrow \quad 0 = -2ax + a^2$

$\Rightarrow \quad x = \frac{a}{2}$

(B) Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, 0)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.



Then, $PA = PB = PC = OP$

Now, $OP = PA \Rightarrow OP^2 = PA^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + (y - b)^2 + (z - 0)^2$

$\Rightarrow x^2 + y^2 + z^2 = x^2 + y^2 + b^2 - 2by + z^2$

$\Rightarrow b^2 = 2by$

$\Rightarrow \quad y = \frac{b}{2}$

(C) *(c)* $\frac{c}{2}$

Explanation: Let $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$ and $O(0, 0, 0)$ be four points equidistant from the point $P(x, y, z)$.

Then, $PA = PB = PC = OP$

Now, $OP = PA$

$\Rightarrow \quad OP^2 = PA^2$

$\Rightarrow \quad x^2 + y^2 + z^2 = x^2 + y^2 + (z - c)^2$

$\Rightarrow \quad x^2 + y^2 + z^2 = x^2 + y^2 + z^2 + c^2 - 2zc$

$\Rightarrow \quad 0 = -2zc + c^2$

$\Rightarrow \quad z = \frac{c}{2}$

(D) *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: Distance formula is used to find the coordinates of point. And points (2, 3, 4) lies in the I octant since, all the coordinaes are positive.

(E) *(a) (0, 0, 0)*

Explanation: Coordinates of origin is (0, 0, 0).

OBJECTIVE Type Questions

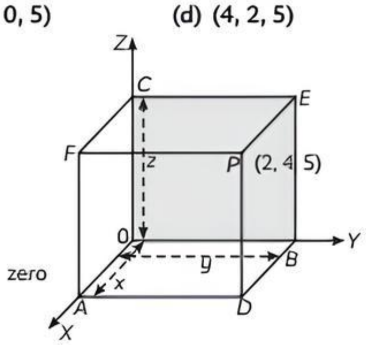
[1 mark]

Multiple Choice Questions

1. In the given figure, if P is (2, 4, 5), the coordinates of F is:

(a) (2, 4, 5) (b) (4, 0, 5)

(c) (2, 0, 5) (d) (4, 2, 5)



Ans. *(c) (2, 0, 5)*

Explanation: For the point F, the distance measured along OY is zero. Therefore, the coordinates of F are (2, 0, 5).

2. The octants in which the points (–3.5, 1.5, 2.5) and (–3.2, 1.2, –2.2) lie are respectively:

(a) Second, fourth (b) Sixth, second

(c) Fifth, sixth (d) Second, sixth

Ans. *(d) Second, sixth*

Explanation: The point (–3.5, 1.5, 2.5) lies in second octant $(-x, y, z)$ and the point (–3.2, 1.2, –2.2) lies in sixth octant $(-x, y, -z)$.

3. Let L, M, N be the feet of the perpendiculars drawn from a point $P(7.2, 9.3, 3.4)$ on the x, y and z-axes respectively. Find the coordinates of L, M and N respectively.

(a) (7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)
(b) (7, 0, 0), (0, 0, 9), (0, 4, 0)
(c) (0, 7, 0), (0, 0, 9), (4, 0, 0)
(d) (0, 0, 7), (0, 9, 0), (4, 0, 0)

Ans. *(a) (7.2, 0, 0), (0, 9.3, 0), (0, 0, 3.4)*

Explanation: Since *L* is the foot of perpendicular from *P* on the *x*-axis, so its *y* and *z*-coordinates are zero. So, the coordinates of *L* is (7.2, 0, 0). Similarly, the coordinates of *M* and *N* are (0, 9.3, 0) and (0, 0, 3.4), respectively.

4. **The point *A*(–4, –3, –2) is present in:**
(a) V-octant (b) VI-octant
(c) VII-octant (d) VIII-octant

[Diksha]

Ans. *(c) VII-octant*

Explanation: The point *A*(–4, –3, –2) lies in VII octant.

5. **What is the perpendicular distance of the point *P*(5, 6, 7) from *xy*-plane?**
(a) 8 units (b) 7 units
(c) 6 units (d) 5 units

Ans. *(b) 7 units*

Explanation: Let *L* be the foot of perpendicular drawn from the point *P*(5, 6, 7) on the *xy*-plane, i.e. $z = 0$

Then coordinates of $L = (5, 6, 0)$

∴ Required distance *PL*

$$= \sqrt{(5-5)^2 + (6-6)^2 + (7-0)^2} = 7 \text{ units}$$

6. **Let *A*, *B*, *C* be the feet of the perpendicular segments drawn from a point *P*(1, 2, 5) on the *xy*, *yz* and *zx*-planes, respectively. The distance of the points *A*, *B*, *C* from the point *P* (in units) respectively are:**
(a) 5, 2, 4 (b) 3, 4, 5
(c) 5, 1, 4 (d) 3, 5, 4

Ans. *(c) 5, 1, 4*

Explanation: We have, coordinates of $A = (1, 2, 0)$, coordinates of $B = (0, 2, 5)$, coordinates of $C = (1, 0, 5)$

Now, $P = (1, 2, 5)$

$$\therefore \quad PA = \sqrt{(1-1)^2 + (2-2)^2 + (5-0)^2} = 5 \text{ units}$$

$$PB = \sqrt{(1-0)^2 + (2-2)^2 + (5-5)^2} = 1 \text{ units}$$

$$PC = \sqrt{(1-1)^2 + (2-0)^2 + (5-5)^2} = 4 \text{ units}$$

7. **The distance between (3, 2, –1) and (–1, –1, –1) is:**
(a) 5 units (b) 6 units
(c) 7 units (d) 8 units [Diksha]

Ans. *(a) 5 units*

Explanation: The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Two points are (3, 2, –1) and (–1, –1, –1), then

$$PQ = \sqrt{(3+1)^2 + (2+1)^2 + (-1+1)^2}$$

$$= \sqrt{16+9+0} = \sqrt{25}$$

$$= 5 \text{ units}$$

8. **Find the distance between the points *P*(0, 3, 4) and *Q*(4, 1, 0).**
(a) 5 units (b) 4 units
(c) 6 units (d) 2 units

Ans. *(c) 6 units*

Explanation: The distance between the points *P*(0, 3, 4) and *Q*(4, 1, 0) is

$$PQ = \sqrt{(4-0)^2 + (1-3)^2 + (0-4)^2}$$

$$= \sqrt{16+4+16} = \sqrt{36} = 6 \text{ units}$$

9. **A point on *ZX*-plane which is equidistant from the points (1, –1, 0), (2, 1, 2), (3, 2, –1) is:**

(a) $\left(\frac{1}{5}, 0, \frac{31}{10}\right)$ (b) $\left(\frac{11}{2}, 0, 1\right)$

(c) $\left(\frac{31}{10}, 0, \frac{1}{5}\right)$ (d) $\left(\frac{31}{5}, 0, \frac{1}{10}\right)$

[Delhi Gov. QB 2022]

Ans. *(b)* $\left(\frac{11}{2}, 0, 1\right)$

Explanation: We know that *y*-coordinate of every point on the *ZX*-plane is zero.

So, let P(*x*, 0, *z*) be a point on the *ZX*-plane such the $PA = PB = PC$

Now, $PA = PB$

$$\Rightarrow (x-1)^2 + (0+1)^2 + (z-0)^2 = (x-2)^2 + (0-1)^2 + (z-2)^2 \; 4x$$

$$\Rightarrow x^2 + 1 - 2x + 1 + z^2 = x^2 - 4 + 4 + 1 + z^2 - 4z + 4$$

$$\Rightarrow -2x + 2 = -4x - 4z + 9$$

$$\Rightarrow -2x + 4x - 4z = 7$$

$$\Rightarrow 2x - 4z = 7$$

$$\Rightarrow x - 2z = \frac{7}{2} \quad \text{...(i)}$$

Now, $PB = PC$

$$\Rightarrow PB^2 = PC^2$$

$$\Rightarrow (x-2)^2 + (0-1)^2 + (z-2)^2 = (x-3)^2 + (0-2)^2 + (z+1)^2$$

$$\Rightarrow z^2 - 4x + 4 + 1 + z^2 - 4z + 4 = x^2 + 9 - 6x + 4 + z^2 + 1 + 2z$$

$\Rightarrow \quad -4z - 4x + 9 = -6x + 2z + 14$

$\Rightarrow \quad -4x + 6x - 4z - 2z = 14 - 9$

$\Rightarrow \quad 2x - 6z = 5$

$\Rightarrow \quad x - 3z = \frac{5}{2}$

$\therefore \quad x = \frac{5}{2} + 3z \quad$ ...(ii)

Putting the value of x in equation (i) :

$$x - 2z = \frac{7}{2}$$

$\Rightarrow \quad \frac{5}{2} + 3z - 2z = \frac{7}{2}$

$\Rightarrow \quad \frac{5}{2} + z = \frac{7}{2}$

$\Rightarrow \quad z = \frac{7}{2} - \frac{5}{2}$

$\Rightarrow \quad z = \frac{7-5}{2}$

$\Rightarrow \quad z = \frac{2}{2}$

$\therefore \quad z = 1$

Putting the value of z in equation (ii):

$$x = \frac{5}{2} + 3z$$

$\Rightarrow \quad x = \frac{5}{2} + 3(1)$

$\Rightarrow \quad x = \frac{5}{2} + 3$

$\Rightarrow \quad x = \frac{5+6}{2}$

$$x = \frac{11}{2}$$

Hence, the required point is $\left(\frac{11}{2}, 0, 1\right)$.

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of thefollowing choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

10. Assertion (A): The points $A(1, -1, 3)$, $B(2, -4, 5)$ and $C(5, -13, 11)$ are collinear.

Reason (R): If $AB + BC = AC$, then A, B, C are collinear.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: Given three points are $A(1, -1, 3)$, $B(2, -4, 5)$ and $C(5, -13, 11)$

$$|AB| = \sqrt{(1)^2 + (-3)^2 + (2)^2}$$

[Using distance formula]

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

$$|BC| = \sqrt{(3)^2 + (-9)^2 + (6)^2}$$

$$= \sqrt{9+81+36}$$

$$= 3\sqrt{14}$$

$$|AC| = \sqrt{(4)^2 + (-12)^2 + (8)^2}$$

$$\sqrt{16+144+64} = 4\sqrt{14}$$

$\because \quad AB + BC = \sqrt{14} + 3\sqrt{14}$

$$= 4\sqrt{14} = AC$$

We know that three points A, B and C are said to be collinear, if

$$AB + BC = AC$$

$\therefore$ Points A, B and C are collinear.

11. Assertion (A): Coordinates of centroid of a triangle formed by the vertices $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(0, 2, 4)$ are $\left(\frac{8}{3}, \frac{8}{3}, \frac{8}{3}\right)$.

Reason (R): Coordinates of centroid of a triangle with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is,

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Ans. *(d) (A) is false but (R) is true.*

Explanation: Coordinates of centroid of a triangle with vertices $A(3, 2, 0)$, $B(5, 3, 2)$ and $C(0, 2, 4)$ are

$$\left(\frac{3+5+0}{3}, \frac{2+3+2}{3}, \frac{0+2+4}{3}\right) = \left(\frac{8}{3}, \frac{7}{3}, 2\right).$$

12. Assertion (A): The foot of perpendicular drawn from the point $A(1, 2, 8)$ on the xy-plane is $(1, 2, 0)$.

Reason (R): Equation of xy-plane is $y = 0$.

Ans. *(c) (A) is true but (R) is false.*

Explanation: We know that in *xy*-plane, *z*-coordinate is 0. So, coordinate of foot of perpendicular drawn from point $A(1, 2, 8)$ on *xy*-plane is $(1, 2, 0)$.

13. **Assertion (A):** The distance between the points $(1+\sqrt{11}, 0, 0)$ and $(1, -2, 3)$ is $2\sqrt{6}$ units.

Reason (R): Distance between any two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is,

$$|AB| = \sqrt{(x_2+x_1)^2+(y_2+y_1)^2+(z_2+z_1)^2}$$

Ans. *(c) (A) is true but (R) is false.*

Explanation: Let the points $A\ (1+\sqrt{11}, 0, 0)$ and $B(1, -2, 3)$

The distance between the points *A* and *B*

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$\therefore\ AB = \sqrt{(1-1-\sqrt{11})^2+(-2-0)^2+(3-0)^2}$$

$$= \sqrt{11+4+9} = \sqrt{24} = 2\sqrt{6} \text{ units}$$

14. **Assertion (A):** The points $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$ and $D(1, -2, 8)$ are the vertices of a parallelogram.

Reason (R): Coordinates of mid-point of a line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right).$$

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: The given point are $A(3, -1, 2)$, $B(1, 2, -4)$, $C(-1, 1, 2)$ and $D(1, -2, 8)$.

$$\text{Mid-point of } AC = \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$$

$$= (1, 0, 2)$$

$$\text{Mid-point of } BD = \left(\frac{1+1}{2}, \frac{2-2}{2}, \frac{-4+8}{2}\right) = (1, 0, 2)$$

$\because$ Mid-points of *AC* and *BD* coincides.

$\therefore$ *ABCD* is a parallelogram.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

15. *Pankaj and his father were walking in a large park. They saw a kite flying in the sky. The position of Kite, Pankaj and Pankaj's father are at (20, 30, 10), (4, 3, 7) and (5, 3, 7) respectively.*

(A) The distance between Pankaj and Kite is:

(a) 41.32 units (b) 31.52 units

(c) 38.32 units (d) 40.39 units

(B) The distance between Pankaj's father and kite is:

(a) 31.30 units (b) 38.43 units

(c) 31.03 units (d) 29.00 units

(C) The co-ordinates of Pankaj lie in:

(a) IV quadrant (b) III quadrant

(c) II quadrant (d) I quadrant

(D) If co-ordinate of Kite, Pankaj and Pankaj's father form a triangle, then the centroid is:

(a) (9.67, 12, 8) (b) (9.6, 8, 12)

(c) (12, 8, 10) (d) (7, 9, 7.2)

(E) The co-ordinates of points in the XY-plane are of the form:

(a) (0, 0, z) (b) (x, y, 0)

(c) (x, 0, y) (d) (0, x, y)

Ans. (A) *(b) 31.52 units*

Explanation: Required distance

$$= \sqrt{(20-4)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{16^2+27^2+3^2}$$

$$= \sqrt{256+729+9}$$

$$= \sqrt{994}$$

$$= 31.52 \text{ units}$$

(B) *(c) 31.03 units*

Explanation: Required distance

$$= \sqrt{(20-5)^2+(30-3)^2+(10-7)^2}$$

$$= \sqrt{15^2+27^2+3^2}$$

$= \sqrt{255+729+9}$

$= \sqrt{963}$

= 31.03 units

(C) *(d) I quadrant*

Explanation: Because in (4, 3, 7); all are positive.

Thus, the coordinate lies in the I quadrant.

(D) *(a) (9.67, 12, 8)*

Explanation: Centroid

$$= \left(\frac{20+4+5}{3}, \frac{30+3+3}{3}, \frac{10+7+7}{3}\right)$$

= (9.67, 12, 8)

(E) *(b) (x, y, 0)*

Explanation: For *XY*-plane, $z = 0$

$\Rightarrow$ The co-ordinates are of the form $(x, y, 0)$.

16. *Vikas and his friends went camping for 2 nights and 3 days. There they set up a tent which is triangular in shape. The vertices of the tent are A(4, 5, 9), B(3, 2, 5), C(5, 2, 5), D(–3, 2, –5) and E(–4, 5, –9) respectively.*

The vertex A is tied up by the rope at the ends F and G and the vertex E is tied up at the ends I and H.

(A) **If *M* denotes the position of their bags inside the tent and it is just in middle of the vertices *B* and *D*, then find the coordinates of *M* and the length *AE*.**

(B) **If the length of the rope by which *E* is tied up with *H* is $5\sqrt{2}$ units, then find the equation denotes the set of point of *H* and the length *BC*.**

(C) **Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).**

Ans. (A) As, *M* is the middle point of *B*(3, 2, 5) and *D*(–3, 2, –5)

$\therefore$ The coordinates of *M* are

$$\left(\frac{3-3}{2}, \frac{2+2}{2}, \frac{5-5}{2}\right) = (0, 2, 0)$$

The length *AE* is

$$= \sqrt{(-4-4)^2+(5-5)^2+(-9-9)^2}$$

$$= \sqrt{64+0+324}$$

$$= \sqrt{388}$$

$$= 2\sqrt{97} \text{ units}$$

(B) As, the distance of $H(x, y, z)$ from $E(-4, 5, -9)$ is $5\sqrt{2}$ units.

$\therefore$ $EH = 5\sqrt{2}$

$$\Rightarrow \sqrt{(x+4)^2+(y-5)^2+(z+9)^2} = 5\sqrt{2}$$

On squaring both sides, we get

$$(x+4)^2 + (y-5)^2 + (z+9)^2 = 25 \times 2$$

$$x^2 + y^2 + z^2 + 8x - 10y + 18z + 122 = 50$$

$$\Rightarrow x^2 + y^2 + z^2 + 8x - 10y + 18z + 72 = 0$$

The length *BC* is,

$$BC = \sqrt{(5-3)^2+(2-2)^2+(5-5)^2}$$

$$= \sqrt{4+0+0}$$

= 4 units

(C) Assume that $P(x, y, z)$ be the point that is equidistant from two points $A(1, 2, 3)$ and $B(3, 2, -1)$.

Thus, we can say that, $PA = PB$

Take square on both the sides, we get

$$PA^2 = PB^2$$

It means that,

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$$

Now, simplify the above equation, we get

$$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$$

$$\Rightarrow -2x - 6z + 6x - 2z = 0$$

$$\Rightarrow 4x - 8z = 0$$

$$\Rightarrow x - 2z = 0$$

Hence, the required equation for the set of points is $x - 2z = 0$.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

17. Find the length of the perpendicular drawn from the point $P(2, 3, 4)$ on y-axis.

Ans. Given point is $P(2, 3, 4)$.

Let Q be the foot of perpendicular drawn from the point $P(2, 3, 4)$ on y-axis.

Then the coordinates of Q are $(0, 3, 0)$

Hence, the length of the perpendicular is given by

$$PQ = \sqrt{(0-2)^2+(3-3)^2+(0-4)^2}$$

$$= \sqrt{4+0+16} = \sqrt{20} \text{ units}$$

18. Find the perpendicular distance of the point $P(-3, 4, -5)$ from z-axis. [Delhi Gov. QB 2022]

Ans. Perpendicular distances from any point (a, b, c) to z-axis will be suppose

$$h = \sqrt{a^2+b^2}$$

$$\therefore (-3, 4, 5) \text{ from } z\text{-axis} = \sqrt{(-3)^2+(4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25} = 5 \text{ units}$$

19. Using distance formula, check wether the points $A(0, 3, 5)$, $B(1, 0, 3)$ and $C(7, 0, 1)$ are collinear or not.

Ans. Given points are $A(0, 3, 5)$, $B(1, 0, 3)$ and $C(7, 0, 1)$.

Then, $AB = \sqrt{(1-0)^2+(0-3)^2+(3-5)^2}$

$$= \sqrt{1+9+4} = \sqrt{14}$$

$$BC = \sqrt{(7-1)^2+(0-0)^2+(1-3)^2}$$

$$= \sqrt{36+0+4} = \sqrt{40} = 2\sqrt{10}$$

$$AC = \sqrt{(7-0)^2+(0-3)^2+(1-5)^2}$$

$$= \sqrt{49+9+16} = \sqrt{74}$$

Since, $AB + BC$ is not equal to AC.

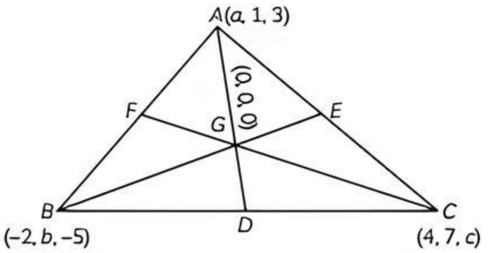
Hence, the points A, B and C are not collinear.

20. If the origin is the centroid of a ΔABC having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, then find values of a, b, c. [NCERT Exemplar]

Ans. Given that origin is the centroid of the ΔABC i.e., $G(0, 0, 0)$.

Centroid of ABC is

$$\because \left(\frac{a-2+4}{3}\right)\left(\frac{1+b+7}{3}\right), \left(\frac{3-5+c}{3}\right).$$



$\because$ Centroid of ABC is $\left(\frac{a+2}{3}, \frac{b+8}{3}, \frac{c-2}{3}\right)$

$\therefore$ Centroid is $(0, 0, 0)$ then

$$\frac{a+2}{3}=0, \frac{b+8}{3}=0, \frac{c-2}{3}=0$$

$\therefore$ $a = -2$, $b = -8$ and $c = 2$

21. Show that, if $x^2 + y^2 = 1$, then the point $(x, y, \sqrt{1-x^2-y^2})$ is at a distance of 1 unit from the origin. [NCERT Exemplar]

Ans. Given that, $x^2 + y^2 = 1$

The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$\therefore$ Distance of the point $(x, y, \sqrt{1-x^2-y^2})$ from origin is given as

$$PQ = \sqrt{(x-0)^2+(y-0)^2+(\sqrt{1-x^2-y^2}-0)^2}$$

$$= \sqrt{x^2+y^2+1-x^2-y^2}$$

$$= 1$$

Hence, proved.

22. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13, 10 and 8 unit. [Delhi Gov. QB 2022]

Ans. Since given dimensions are $a = 10$, $b = 13$ and $c = 8$

$\therefore$ Required length of the string is $\sqrt{a^2+b^2+c^2}$

i.e., $\sqrt{(10)^2+(13)^2+8^2}$

i.e., $\sqrt{100+169+64} = \sqrt{333}$ units

i.e., the length of the longest piece of a string that can be stretched straight in a rectangular room with dimensions 10, 13 and 8 are $\sqrt{333}$ units.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

23. Find the point on *x*-axis which is equidistant from the points *A*(0, 3, 2) and *B*(5, 0, 4).

Ans. Let *P*(*x*, 0, 0) be the point on *x*-axis which is equidistant from the point *A*(0, 3, 2) and *B*(5, 0, 4).

$\therefore \quad AP = BP$

$\Rightarrow \sqrt{(x-0)^3+(0-3)^2+(0-2)^2}$

$= \sqrt{(x-5)^2+(0-0)^2+(0-4)^2}$

On squaring both sides, we get

$(x-0)^2 + 9 + 4 = (x-5)^2 + 0 + 16$

$\Rightarrow \quad x^2 + 13 = x^2 + 25 - 10x + 16$

$\Rightarrow \quad 10x = 28$

$\Rightarrow \quad x = \frac{14}{5}$

Hence, the required point is $\left(\frac{14}{5}, 0, 0\right)$.

24. If the distance between the points (*a*, 2, 1) and (1, –1, 1) is 5, then find the sum of all possible value of *a*. [Delhi Gov. QB 2022]

Ans. The distance between two points

$P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

The distance between two points (*a*, 2, 1) and (1, –1, 1) is given as 5.

$\sqrt{(a-1)^2+(2+1)^2+(1-1)^2} = 5$

$\Rightarrow \quad (a-1)^2 + 9 = 25$

$\Rightarrow \quad (a-1)^2 = 16$

$\Rightarrow \quad a - 1 = 4 \text{ or } a - 1 = -4$

$a = 5 \text{ and } -3$

Hence, the sum of all possible values of $a = 5 + (-3) = 2$

25. Find the equation of the set of the points *P* such that its distances from the points *A*(2, 0, –5) and *B*(0, 1, 4) are equal.

Ans. Given points are *A*(2, 0, –5) and *B*(0, 1, 4).

Let *P*(*x*, *y*, *z*) be any point such that its distance from the points *A* and *B* are equal.

$\therefore \quad AP = BP$

$\Rightarrow \sqrt{(x-3)+(y-0)^2+(z+5)^2}$

$= \sqrt{(x+0)^2+(y-1)^2+(z-4)^2}$

On squaring both sides, we get

$(x-3)^2 + (y)^2 + (z+5)^2 = (x)^2 + (y-1)^2 + (z-4)^2$

$\Rightarrow \quad -6x + 2y + 18z + 17 = 0$

which is the required equation.

26. Show that the points *A*(1, –1, 3), *B*(2, –4, 5) and *C*(5, –13, 11) are collinear. [NCERT Exemplar]

Ans. Given points are *A*(1, –1, 3), *B*(2, –4, 5) and *C*(5, –13, 11)

$AB = \sqrt{(1-2)^2+(-1+4)^2+(3-5)^2}$

$= \sqrt{1+9+4} = \sqrt{14}$

$BC = \sqrt{(2-5)^2+(-4+13)^2+(5-11)^2}$

$= \sqrt{9+18+36} = \sqrt{126} = 3\sqrt{14}$

$AC = \sqrt{(1-5)^2+(-1+13)^2+(3-11)^2}$

$= \sqrt{16+144+64} = \sqrt{224} = 4\sqrt{14}$

If *AB* + *BC* = *AC*, the *A*, *B*, *C* are collinear.

$\Rightarrow \quad \sqrt{14} + \sqrt{126} = \sqrt{224}$

$\Rightarrow \quad \sqrt{14} + 3\sqrt{14} = 4\sqrt{14}$

So, the points *A*, *B* and *C* are collinear.

27. If *A*, *B*, *C* be the feet of perpendicular from a point *P* on the *X*, *Y* and *Z*-axes respectively, then find the coordinates of *A*, *B* and *C* in each of the following where the point *P* is:

(A) (3, 4, 2)

(B) (–5, 3, 7)

(C) (4, –3, –5) [NCERT Exemplar]

Ans. We know that, on *x*-axis, *y*, *z* = 0, on *y*-axis *x*, *z* = 0 and on *z*-axis, *x*, *y* = 0. Thus, the feet of perpendiculars from given point *P* on the axis are of follows:

(A) *A*(3, 0, 0), *B*(0, 4, 0), *C*(0, 0, 2)

(B) *A*(–5, 0, 0), *B*(0, 3, 0), *C*(0, 0, 7)

(C) *A*(4, 0, 0), *B*(0, –3, 0), *C*(0, 0, –5)

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

28. If the distance between the points (a, 0, 2) and (1, 1, 0) is $\sqrt{14}$ units, then find the value of a.

Ans. Let the given points be $A(a, 0, 2)$ and $B(1, 1, 0)$.

Given that the distance between A and B is 5.

$\therefore \quad AB = 5$

$\Rightarrow \quad \sqrt{(1-a)^2+(1-0)^2+(0-2)^2} = \sqrt{14}$

$\Rightarrow \quad (1-a)^2 + 1 + 4 = 14$

$\Rightarrow \quad 1 + a^2 - 2a + 5 = 14$

$\Rightarrow \quad a^2 - 2a - 8 = 0$

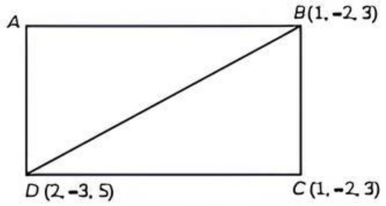
$\Rightarrow \quad (a+4)(a-2) = 0$

$\Rightarrow \quad a = -4, 2.$

29. If the extremities (end points) of a diagonal of a square are (1, –2, 3) and (2, –3, 5) then find the length of the side of square.

[Delhi Gov. QB 2022]

Ans.



The points are $B(1, -2, 3)$ and $D(2, -3, 5)$

$\therefore \quad BD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(2-1)^2+(-3+2)^2+(5-3)^2}$

$= \sqrt{1+1+4}$

$= \sqrt{6}$

We know that the length of diagonal is equal to $\sqrt{2}$ times of side length.

$\therefore \sqrt{6} = \sqrt{2} \times$ side length of square

Length of side of square $= \dfrac{\sqrt{6}}{\sqrt{2}}$

$= \sqrt{\dfrac{6}{2}}$

$= \sqrt{3}$

30. If the origin is the centroid of ΔPQR with vertices $P(a, 0, 6)$, $Q(4, b, -1)$and $R(2, 4, c)$, then find the values of a, b and c.

Ans. Given, vertices of ΔPQR are $P(a, 0, 6)$, $Q(4, b, -1)$ and $R(2, 4, c)$.

Then, the coordinates of the centroid of ΔPQR are given by

$$\left(\frac{a+4+2}{3}, \frac{0+b+4}{3}, \frac{6-1+c}{3}\right)$$

i.e., $\left(\dfrac{a+6}{3}, \dfrac{b+4}{3}, \dfrac{c+5}{3}\right)$

Given, that the centroid of ΔPQR is the point (0, 0, 0).

$\therefore \quad \dfrac{a+6}{3} = 0$

$\Rightarrow \quad a = -6$

$\dfrac{b+4}{3} = 0$

$\Rightarrow \quad b = -4$

$\dfrac{c+5}{3} = 0$

$\Rightarrow \quad c = -5$

Hence, $a = -6$, $b = -4$ and $c = -5$.

31. Show that the points $A(1, 2, 3)$, $B(-1, -2, -1)$, $C(2, 3, 2)$ and $D(4, 7, 6)$ are the vertices of parallelogram ABCD but it is not a rectangle.

[Delhi Gov. Term-2 SQP 2022]

Ans. To show $ABCD$ is a parallelogram we need to show opposite sides are equal note that

$AB = \sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}$

$= \sqrt{4+16+16} = 6$

$BC = \sqrt{(2+1)^2+(3+2)^2+(2+1)^2}$

$= \sqrt{9+25+9} = \sqrt{43}$

$CD = \sqrt{(4-2)^2+(7-3)^2+(6-2)^2}$

$= \sqrt{4+16+16} = 6$

$DA = \sqrt{(1-4)^2+(2-7)^2+(3-6)^2}$

$= \sqrt{9+25+9} = \sqrt{43}$

Since, $AB = CD$ and $BC = AD$, $ABCD$ is a parallelogram.

Now, it is required to prove that $ABCD$ is not a rectangle. For this we need to show that diagonals are unequal, we have

$AC = \sqrt{(2-1)^2+(3-2)^2+(2-3)^2}$

$= \sqrt{1+1+1} = \sqrt{3}$

$BD = \sqrt{(4+1)^2+(7+2)^2+(6+1)^2}$

$= \sqrt{25+81+49} = \sqrt{155}$

Since, $AC \neq BD$, $ABCD$ is not a rectangle.

32. **If *A*, *B* and *C* be the feet of perpendicular from a point *P* on the *XY*, *YZ* and *ZX*-planes respectively, then find the coordinates of *A*, *B* and *C* in each of the following where the point *P* is:**

(A) (3, 4, 5)

(B) (–5, 3, 7)

(C) (4, –3, –5) **[NCERT Exemplar]**

Ans. We know that, on *XY*-plane $Z = 0$, on *YZ*-plane, $x = 0$ and on *ZX*-plane, $y = 0$. Thus, the coordinates of feet of perpendicular on the *xy*, *yz* and *zx*-planes from the given point are as follows:

(A) $A(3, 4, 0)$, $B(0, 4, 5)$, $C(3, 0, 5)$

(B) $A(-5, 3, 0)$, $B(0, 3, 7)$, $C(-5, 0, 7)$

(C) $A(4, -3, 0)$, $B(0, -3, -5)$, $C(4, 0, -5)$

33. **Find the point *P* on *z*-axis such that *PA* = *PB* where *A*(1, 5, 7), *B*(5, 1, –4).** **[Diksha]**

Ans. Given that,

$$A = (1, 5, 7) \text{ and } B = (5, 1, -4)$$

To find:

A point at *z*-axis, equidistant from *A* and *B*.

Consider the point be *P* having co-ordinates , as it is on *z*-axis.

According to the question,

$$\overline{PA} = \overline{PB} \text{ on, } (PA)^2 = (PB)^2$$

$$\Rightarrow (1-0)^2 + (5-0)^2 + (7-a)^2 = (5-0)^2 + (1-0)^2 + (-4-a)^2$$

$$\Rightarrow 1 + 25 + 49 + a^2 - 14a = 25 + 1 + 16 + a^2 + 8a$$

$$\Rightarrow 14a + 8a = 49 - 16$$

$$\Rightarrow 22a = 33$$

$$\Rightarrow a = \frac{3}{2}$$

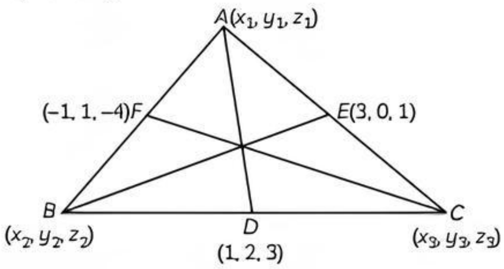
Therefore, point '*p*' is $\left(0, 0, \frac{3}{2}\right)$.

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

34. **Find the centroid of a triangle, the mid-point of whose sides are *D*(1, 2, –3), *E*(3, 0, 1) and *F*(–1, 1, –4).**

Ans. Given that, mid-point of sides are $D(1, 2, -3)$ and $F(-1, 1, -4)$.



Let the vertices of the ΔABC are

$A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$

Then, mid-point of *BC* are (1, 2, –3).

$$\therefore \quad 1 = \frac{x_2 + x_3}{2}$$

$$\Rightarrow x_2 + x_3 = 2 \quad \text{...(i)}$$

$$2 = \frac{y_2 + y_3}{2}$$

$$\Rightarrow y_2 + y_3 = 4 \quad \text{...(ii)}$$

$$-3 = \frac{z_2 + z_3}{2}$$

$$\Rightarrow z_2 + z_3 = -6 \quad \text{...(iii)}$$

Now the mid-points of *AC* and *AB*

$$\Rightarrow -1 = \frac{x_1 + x_2}{2}$$

$$\Rightarrow x_1 + x_2 = -2 \quad \text{...(iv)}$$

$$\Rightarrow 1 = \frac{y_1 + y_2}{2}$$

$$\Rightarrow y_1 + y_2 = 2 \quad \text{...(v)}$$

$$\Rightarrow -4 = \frac{z_1 + z_2}{2}$$

$$\Rightarrow z_1 + z_2 = -8 \quad \text{...(vi)}$$

$$\Rightarrow 3 = \frac{x_1 + x_3}{2}$$

$$\Rightarrow x_1 + x_3 = 6 \quad \text{...(vii)}$$

$$\Rightarrow 0 = \frac{y_1 + y_3}{2}$$

$$\Rightarrow y_1 + y_3 = 0 \quad \text{...(viii)}$$

$$\Rightarrow 1 = \frac{z_1 + z_3}{2}$$

$$\Rightarrow z_1 + z_3 = 2 \quad \text{...(ix)}$$

On adding eqs.(i) and (iv), we get

$$x_1 + 2x_2 + x_3 = 0 \quad \text{...(x)}$$

On adding eqs. (ii) and (v), we get

$$y_1 + 2y_2 + y_3 = 6 \quad \text{...(xi)}$$

On adding eqs. (iii) and (vi), we get

$$z_1 + 2z_2 + z_3 = -14 \quad ...(xii)$$

From eqn. (vii) and (x),

$$2x_2 = -6$$

$$\Rightarrow \quad x_2 = -3$$

If $x_2 = -3$, then $x_3 = 5$

If $x_3 = 5$, then $x_1 = 1, x_2 = -3, x_3 = 5$

From eqn. (xi) and (viii)

$$2y_2 = 6$$

$$\Rightarrow \quad y_2 = 3$$

If $y_2 = 3$, then $y_1 = -1$ if y_1

$= -1$, then $y_3 = 1, y_2 = 3, y_3 = 1$

From eqn. (xii) and (ix)

$$2z_2 = -16$$

$$\Rightarrow \quad z_2 = -8$$

$z_2 = -8$, then $z_1 = 2$

$z_1 = 0$, then $z_3 = 0$

$z_1 = 0, z_2 = -8, z_3 = 2$

So, the points are $A(1, -1, 0)$, $B(-3, 3, -8)$ and $C(5, 1, 2)$.

$\therefore$ Centroid of the triangle

$$= G\left(\frac{1-3+5}{3}, \frac{-1+3+1}{3}, \frac{0-8+2}{3}\right)$$

i.e., $G(1, 1, -2)$.

35. If a parallelepiped is formed by planes drawn through the points (5, 8, 10) and (3, 6, 8) parallel to the coordinate planes, then find the length of edges and diagonal of the parallelepiped by using distance formula.

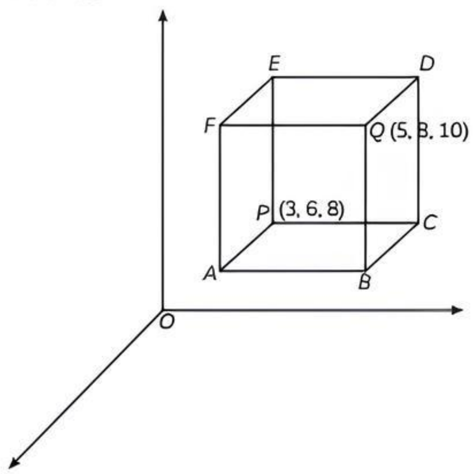
[Delhi Gov. QB 2022]

Ans. Let $P = (3, 6, 8)$ and $Q = (5, 8, 10)$

PE = Distance between parallel lines $ABCP$ and $FQDE$

$= |10 - 8|$ [$\because$ These planes are perpendicular to Z-axis]

$= 2$ units.



PE = Distance between parallel planes $ABQF$ and $PCDE$

$= |5 - 3|$ [$\because$ these planes are perpendicular to X-axis]

$= 2$ units

PC = Distance between parallel planes APEF and BCDQ

$= |8 - 6|$ [$\because$ these planes are perpendicular to Y-Axis]

$= 2$ units

$\therefore$ Length of diagonal = Distance between P and Q.

$$= \sqrt{(5-3)^2 + (8-6)^2 + (10-8)^2}$$

[using the distance formula]

$$= \sqrt{2^2 + 2^2 + 2^2}$$

$$= \sqrt{4+4+4}$$

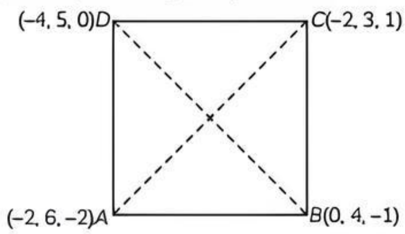
$$= \sqrt{12}$$

$$= 2\sqrt{3} \text{ units}$$

Hence, the length of each edge of parallelepiped is 2 units and the length of its diagonal is $2\sqrt{3}$ units.

36. Show that the points (–2, 6, –2), (0, 4, –1), (–2, 3, 1) and (–4, 5, 0) are the vertices of a square.

Ans. Let $A(-2, 6, -2)$, $B(0, 4, -1)$, $C(-2, 3, 1)$ and $D(-4, 5, 0)$ be the given points.



$$\therefore \quad AB = \sqrt{(0+2)^2 + (4-6)^2 + (-1+2)^2}$$

[using the distance formulas]

$$= \sqrt{4+4+1} = \sqrt{9}$$

$= 3$ units

$$BC = \sqrt{(-2-0)^2 + (3-4)^2 + (1+1)^2}$$

$$= \sqrt{4+1+4} = \sqrt{9}$$

$= 3$ units

$$CD = \sqrt{(-4+2)^2 + (5-3)^2 + (0-1)^2}$$

$$= \sqrt{4+4+1} = \sqrt{9}$$

$= 3$ units

$$AD = \sqrt{(-4+2)^2+(5-6)^2+(0+2)^2}$$
$$= \sqrt{4+1+4} = \sqrt{9}$$
$$= 3 \text{ units}$$

Here, $AB = BC = CD = DA$.

So, $ABCD$ is a square or a rhombus.

$$\text{Now, } AC = \sqrt{(-2+2)^2+(3-6)^2+(1+2)^2}$$
$$= \sqrt{0+9+9}\ ]$$
$$= \sqrt{18} \text{ units}$$

$$\text{and } BD = \sqrt{(-4-0)^2+(5-4)^2+(0+1)^2}$$
$$= \sqrt{16+1+1}$$
$$= \sqrt{18} \text{ units}$$

Since, diagonal AC = diagonal BD

Hence, $ABCD$ is a square.

37. Show that the triangle ABC with vertices $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$ is right angled.

Ans. Points are $A(0, 4, 1)$, $B(2, 3, -1)$ and $C(4, 5, 0)$

$x_1 = 0$	$x_2 = 2$	$x_3 = 4$
$y_1 = 4$	$y_2 = 3$	$y_3 = 5$
$z_1 = 1$	$y_3 = -1$	$z_3 = 0$

If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are the two points then the distance between these points can be given by the distance formula.

$$PQ = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$\Rightarrow AB = \sqrt{(0-2)^2+(4-3)^2+(1-(-1))^2}$$
$$= \sqrt{(-2)^2+(1)^2+(2)^2}$$
$$= \sqrt{4+1+4}$$
$$= \sqrt{9}$$

$$\Rightarrow BC = \sqrt{(2-4)^2+(3-5)^2+(-1-0)^2}$$
$$BC = \sqrt{(-2)^2+(-2)^2+(-1)^2}$$
$$BC = \sqrt{9}$$
$$BC = 3$$

$$\Rightarrow AC = \sqrt{(0-4)^2+(4-5)^2+(1-0)^2}$$
$$= \sqrt{(-4)^2+(-1)^2+(1)^2}$$
$$= \sqrt{16+1+1}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

In a right-angled triangle, sum of the square of two sides is equal to the square of the largest side.

$$\Rightarrow (AB)^2 + (BC)^2 = (AC)^2$$
$$\Rightarrow (3)^2 + (3)^2 = (3\sqrt{2})^2$$
$$\Rightarrow 9 + 9 = 18$$
$$\Rightarrow 18 = 18$$

Since, $(AB)^2 + (BC)^2 = (AC)^2$

Hence, ΔABC is a right-angled triangle.